Mid-Semester Exam : PROBABILITY II

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Max. points : 10. **Time Limit : 1.5 hours.**

Answer any two questions only. All questions carry 5 points.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class mention it clearly. Always define the underlying events, random variables, pdfs et al. clearly before computing anything !

1. Let $X_1, \ldots, X_n$ be i.i.d. Exponential (λ) random variables for some $\lambda > 0$. Let $X_{(1)} < \ldots < X_{(n)}$ be the corresponding order statistics. Define the *range* $R := X_{(n)} - X_{(1)}$ and *mid-range* $M := \frac{X_{(n)} + X_{(1)}}{2}$. Find the joint pdf of (R, M).

2. Let U_1, U_2 be i.i.d. $U(0,1)$ random variables. Define

   $$X_1 = \sqrt{-2 \log U_1} cos(2\pi U_2), \ X_2 = \sqrt{-2 \log U_1} sin(2\pi U_2).$$

   Find the joint pdf of (X_1, X_2).

3. Let $X, X_1, \ldots, X_n$ be i.i.d. standard Normal random variables. Define $Y := \sqrt{[\sum_{i=1}^{n} X_i^2]/n}$ and $T = X/Y$. Find the joint pdf of (Y, T) and the marginal pdf of T.

PDFs :

- *Exponential (λ) distribution :* $f(x) = \lambda e^{-\lambda x}, x > 0$.

- *Uniform $(0,1)$ distribution :* $f(x) = \mathbf{1}[0 < x < 1]$.

- *Standard Normal distribution :* $f(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}), x \in \mathbb{R}$.